

Problem 2.82

[Difficulty: 2]

2.82 Water usually is assumed to be incompressible when evaluating static pressure variations. Actually it is 100 times more compressible than steel. Assuming the bulk modulus of water is constant, compute the percentage change in density for water raised to a gage pressure of 100 atm. Plot the per-

centage change in water density as a function of p/p_{atm} up to a pressure of 50,000 psi, which is the approximate pressure used for high-speed cutting jets of water to cut concrete and other composite materials. Would constant density be a reasonable assumption for engineering calculations for cutting jets?

Solution: By definition, $E_v = \frac{dp}{d\rho/\rho}$. Assume $E_v = \text{constant}$. Then

$$\frac{dp}{\rho} = \frac{dp}{E_v}$$

Integrating, from p_0 to p gives $\ln \frac{\rho}{\rho_0} = \frac{p - p_0}{E_v} = \frac{\Delta p}{E_v}$, so $\frac{\rho}{\rho_0} = e^{\Delta p/E_v}$

The relative change in density is

$$\frac{\Delta \rho}{\rho_0} = \frac{\rho - \rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = e^{\Delta p/E_v} - 1$$

From Table A.2, $E_v = 2.24 \text{ GPa}$ for water at 20°C .

For $p = 100 \text{ atm (gage)}$, $\Delta p = 100 \text{ atm}$, so

$$\frac{\Delta \rho}{\rho_0} = \exp \left(100 \text{ atm} \times \frac{1}{2.24 \times 10^9 \text{ Pa}} \times \frac{101.325 \times 10^3 \text{ Pa}}{1 \text{ atm}} \right) - 1 = 0.00453, \text{ or } 0.453\%$$

For $\Delta p = 50,000 \text{ psi}$,

$$\frac{\Delta \rho}{\rho_0} = \exp \left(50,000 \text{ psi} \times \frac{1}{2.24 \times 10^9 \text{ Pa}} \times \frac{101.325 \times 10^3 \text{ Pa}}{14.696 \text{ psi}} \right) - 1 = 0.166 \text{ or } 16.6\%$$

Thus constant density is not a reasonable assumption for a cutting jet operating at 50,000 psi. Constant density (5% change) would be reasonable up to $\Delta p \approx 16,000 \text{ psi}$.

